

NOTIZEN

Steady State Film Profiles of Viscous Drainage over a Naturally Permeable Wall

D. Padma *, N. Ch. Pattabhi Ramacharyulu, and V. V. Ramana Rao

International Centre for Theoretical Physics, Trieste, Italy

(Z. Naturforsch. **32a**, 182 [1977];
received November 18, 1976)

The Navier-Stokes equation of viscous drainage over a naturally permeable wall is investigated. It is found that as an effect of porosity, the film falls faster than that in the clear zone.

Recently Annapurna and Ramanaiah¹ have investigated the effect of the inertial contribution to the Jeffreys' solution² corresponding to the steady state film profiles of viscous drainage. In this note we examine the steady state problem and solve it subject to the slip-boundary conditions

$$\left. \frac{\partial V}{\partial x} \right|_{x=0} = \frac{\alpha}{\sqrt{k}} (V_B - V_D), \quad \left. \frac{\partial V}{\partial x} \right|_{x=h} = 0, \quad (1)$$

stipulating that the film surface $x=0$ is permeable as proposed by Beavers and Joseph³ and the film surface $x=h(y)$ stress free. Here k is the permeability of the material, α is a dimensionless quantity depending only on the material parameters, which characterise the structure of the permeable material within the boundary region, V_B is the slip velocity at the permeable surface and V_D is the Darcy velocity given by

$$V_D = -gk/\nu. \quad (2)$$

We then obtain

$$V = -\frac{gh^2}{\nu} \left[\frac{1}{\alpha\sigma} + \frac{1}{\sigma^2} + \frac{x}{h} + \frac{x^2}{2h^2} \right], \quad (3)$$

where $\sigma = h/\sqrt{k}$.

* Department of Applied Mathematics, Andhra University, Waltair, India.

Reprint requests to Dr. V. V. Ramana Rao, Department of Applied Mathematics, Andhra University, Waltair, India.

¹ N. Annapurna and G. Ramanaiah, Z. Naturforsch. **31a**, 1007 [1976].

² H. Jeffreys, Proc. Camb. Phil. Soc. **26**, 204 [1930].

In terms of the non-dimensional quantities defined by Eq. (8) of [1], we obtain

$$y = -H^2 T [1 + (\alpha + 2\sigma)/\alpha\sigma^2]. \quad (4)$$

In the above equation we realize the well-known Jeffreys' solution

$$y = -H^2 T, \quad (5)$$

by allowing $\alpha, \sigma \rightarrow \infty$ independently.

The effect of porosity on the liquid-film-profile is illustrated in Fig. 1, taking $\alpha=1.45$, $\sigma=2$. It is noticed that the film falls much faster than the Jeffreys-film. A physical explanation of this result could be that the effects of viscous shear appear to penetrate into the permeable material in a boundary layer region. This confirms the experimental observation of Beavers and Joseph³.

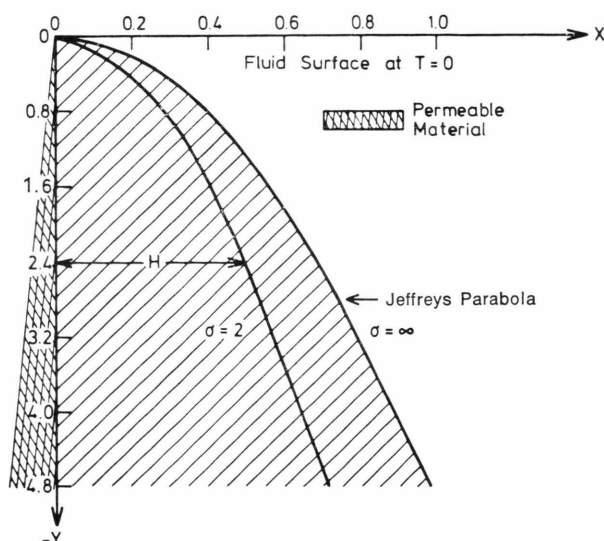


Fig. 1. Steady state film profiles at $T=10$ and $\alpha=1.45$.

Acknowledgement

The authors thank Professor P. G. Drazin, University of Bristol, U.K. for his useful suggestions.



Dieses Werk wurde im Jahr 2013 vom Verlag Zeitschrift für Naturforschung in Zusammenarbeit mit der Max-Planck-Gesellschaft zur Förderung der Wissenschaften e.V. digitalisiert und unter folgender Lizenz veröffentlicht: Creative Commons Namensnennung-Keine Bearbeitung 3.0 Deutschland Lizenz.

Zum 01.01.2015 ist eine Anpassung der Lizenzbedingungen (Entfall der Creative Commons Lizenzbedingung „Keine Bearbeitung“) beabsichtigt, um eine Nachnutzung auch im Rahmen zukünftiger wissenschaftlicher Nutzungsformen zu ermöglichen.

This work has been digitalized and published in 2013 by Verlag Zeitschrift für Naturforschung in cooperation with the Max Planck Society for the Advancement of Science under a Creative Commons Attribution-NoDerivs 3.0 Germany License.

On 01.01.2015 it is planned to change the License Conditions (the removal of the Creative Commons License condition "no derivative works"). This is to allow reuse in the area of future scientific usage.