NOTIZEN

Steady State Film Profiles of Viscous Drainage over a Naturally Permeable Wall

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The Navier-Stokes equation of viscous drainage over a naturally permeable wall is investigated. It is found that as an effect of porosity, the film falls faster than that in the clear zone.

Recently Annapurna and Ramanaiah [1] have investigated the effect of the inertial contribution to the Jeffreys' solution [2] corresponding to the steady state film profiles of viscous drainage. In this note we examine the steady state problem and solve it subject to the slip-boundary conditions

$$\left.\frac{\partial V}{\partial x}\right|_{x=0} = \frac{\alpha}{\sqrt{k}} (V_\mathrm{B} - V_\mathrm{D}), \quad \left.\frac{\partial V}{\partial x}\right|_{x=h} = 0, \tag{1}$$

stipulating that the film surface $x=0$ is permeable as proposed by Beavers and Joseph [3] and the film surface $x=h(y)$ stress free. Here k is the permeability of the material, α is a dimensionless quantity depending only on the material parameters, which characterise the structure of the permeable material within the boundary region, V_B is the slip velocity at the permeable surface and V_D is the Darcy velocity given by

$$V_\mathrm{D} = -g\,k/\nu\,. \tag{2}$$

We then obtain

$$V = -\frac{g\,h^2}{\nu}\left[\frac{1}{\alpha\,\sigma} + \frac{1}{\sigma^2} + \frac{x}{h} + \frac{x^2}{2\,h^2}\right], \tag{3}$$

where $\sigma = h/\sqrt{k}$.

In terms of the non-dimensional quantities defined by Eq. (8) of [1], we obtain

$$y = -H^2\,T[1 + (\alpha + 2\,\sigma)/\alpha\,\sigma^2]\,. \tag{4}$$

In the above equation we realize the well-known Jeffreys' solution

$$y = -H^2\,T\,, \tag{5}$$

by allowing $\alpha, \sigma \to \infty$ independently.

The effect of porosity on the liquid-film-profile is illustrated in Fig. 1, taking $\alpha = 1.45$, $\sigma = 2$. It is noticed that the film falls much faster than the Jeffreys-film. A physical explanation of this result could be that the effects of viscous shear appear to penetrate into the permeable material in a boundary layer region. This confirms the experimental observation of Beavers and Joseph [3].

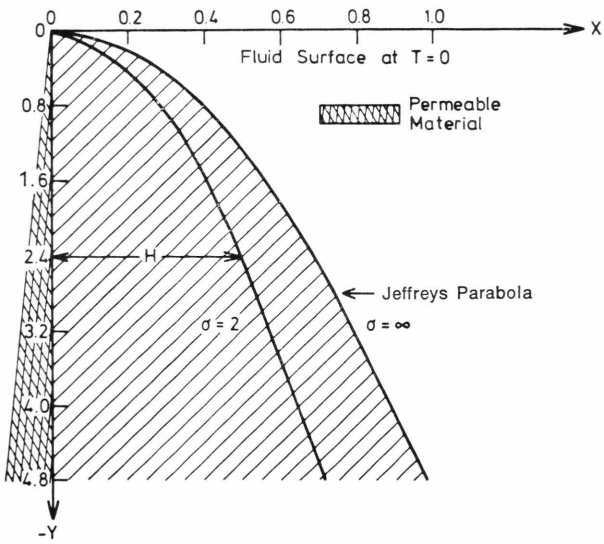

Fig. 1. Steady state film profiles at $T=10$ and $\alpha=1.45$.

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